



PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

(Reproduced from memory retention)

Date : 26 February, 2021 (SHIFT-1) Time ; (9.00 am to 12.00 pm)

Duration : 3 Hours | Max. Marks : 300

SUBJECT : MATHEMATICS

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MATHEMATICS

1. $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx =$

- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{3}$

Ans. (1)

Sol. $I = \int_0^{\pi/2} \left(\frac{\cos^2 x}{1+3^x} + \frac{\cos^2 x}{1+3^{-x}} \right) dx = \int_0^{\pi/2} \left(\frac{\cos^2 x}{1+3^x} + \frac{3^x \cos^2 x}{1+3^x} \right) dx = \int_0^{\pi/2} \cos^2 x dx$
 $= \frac{1}{2} \int_0^{\pi/2} (1 + \cos 2x) dx = \frac{1}{2} \left(x + \frac{1}{2} \sin 2x \right) \Big|_0^{\pi/2} = \frac{1}{2} \left(\frac{\pi}{2} \right) = \frac{\pi}{4}$

2. Value of $\lim_{x \rightarrow 0} 2 \left\{ \frac{\sqrt{3} \sin \left(\frac{\pi}{6} + x \right) - \cos \left(\frac{\pi}{6} + x \right)}{\sqrt{3}x (\sqrt{3} \cos x - \sin x)} \right\}$ is equal to

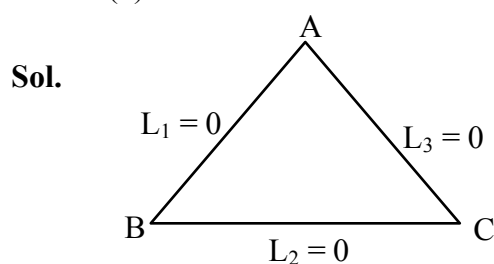
- (1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{2}{3}$ (4) $\frac{4}{\sqrt{3}}$

Ans. (1)

Sol. $= \lim_{x \rightarrow 0} \frac{4 \left[\sin \left(\frac{\pi}{6} + x - \frac{\pi}{6} \right) \right]}{\sqrt{3}x(\sqrt{3})} = \lim_{x \rightarrow 0} \frac{4}{3} \frac{\sin x}{x} = \frac{4}{3}$

3. If $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ are three lines forming a triangle, then the triangle is
 (1) Isosceles (2) Right angled (3) Equilateral (4) None of these

Ans. (1)



$L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : 2x + y = 6$

A (2, 2)

B (1, 1)

C (3, 0)

$\Rightarrow AB = \sqrt{2}, BC = \sqrt{5}, AC = \sqrt{5}$

$\therefore$ Triangle is isosceles

4. Find the number of integral values of k for which the equation $3 \sin x + 4 \cos x = k + 1$ has a solution.

(1) 13 (2) 6 (3) 8 (4) 11

Ans. (4)

Sol. $-\sqrt{3^2 + 4^2} \leq 3 \sin x + 4 \cos x \leq \sqrt{3^2 + 4^2}$
 $-5 \leq (k + 1) \leq 5$
 $-6 \leq k \leq 4$

5. Number of 7 digits number in which sum of digits is 10 and digits can take 1, 2, 3 values, is

(1) 77 (2) 42 (3) 60 (4) 35

Ans. (1)

Sol. **Case-1:** 1, 1, 1, 1, 1, 2, 3

$$\text{ways} = \frac{7!}{5!} = 42$$

Case-2: 1, 1, 1, 1, 2, 2, 2

$$\text{ways} = \frac{7!}{4! \cdot 3!} = 35$$

$$\text{total ways} = 42 + 35 = 77$$

6. Find the number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$

(1) 0 (2) 1 (3) 2 (4) 4

Ans. (2)

Sol. $\log_4(x - 1) = \log_2(x - 3)$
 $\Rightarrow x - 1 = (x - 3)^2 \Rightarrow x = 2, 5$
 but here $x \geq 3$
 $\therefore x = 5$

7. If A is a symmetric matrix of order 2 and sum of diagonal elements of A^2 is 1, where elements of matrix are integer, then number of such matrices are

(1) 4 (2) 6 (3) 8 (4) 5

Ans. (1)

Sol. $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$A^2 = \begin{bmatrix} a^2 + b^2 & b(a + c) \\ b(a + c) & b^2 + c^2 \end{bmatrix}$$

$$\text{tr}(A^2) = a^2 + 2b^2 + c^2 = 1$$

$$\Rightarrow b = 0 \text{ and } a^2 + c^2 = 1$$

$$\Rightarrow (a, c) = (1, 0), (-1, 0), (0, 1), (0, -1)$$

8. The maximum value of slope of tangent to $y = \frac{x^4}{2} - 5x^3 + 18x^2 + 6$ is at a point.
- (1) (2,2) (2) (2,46) (3) $\left(1, \frac{39}{2}\right)$ (4) (1,0)

Ans. (2)

Sol. $m = \frac{dy}{dx} = 2x^3 - 15x^2 + 36x$

$$\frac{dy}{dx} = 6x^2 - 30x + 36$$

$$= 6(x^2 - 5x + 6) = 0$$

$$\Rightarrow x = 2, 3$$

$$\frac{d^2y}{dx^2} = 6(2x - 5)$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=2} = -ve$$

$\therefore$ Maximum at $x = 2$

Point (2, 46)

9. $\{(P, Q) ; P, Q \text{ be 2 points which are equidistant from origin}\}$, then point (x, y) which are equivalence class of $(1, -1)$
- (1) $x^2 + y^2 = 2$ (2) $x^2 + y^2 = \sqrt{2}$ (3) $x^2 + y^2 = 1$ (4) $x^2 + y^2 = 2\sqrt{2}$

Ans. (1)

Sol. The equivalence class containing $(1, -1)$ for this relation is $x^2 + y^2 = 2$

10. Value of $\begin{vmatrix} (a+1)(a+2) & (a+1) & 1 \\ (a+2)(a+3) & (a+2) & 1 \\ (a+3)(a+4) & (a+3) & 1 \end{vmatrix}$ is equal to

- (1) -2 (2) 2 (3) 0 (4) 1

Ans. (1)

Sol. $D = \begin{vmatrix} a^2 + 3a + 2 & a + 1 & 1 \\ a^2 + 5a + 6 & a + 2 & 1 \\ a^2 + 7a + 12 & a + 3 & 1 \end{vmatrix}$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$D = \begin{vmatrix} a^2 + 3a + 2 & a + 1 & 1 \\ 2a + 4 & 1 & 0 \\ 4a + 10 & 2 & 0 \end{vmatrix} = 4a + 8 - 4a - 10 = -2$$

11. If $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$, then find the value of $\cos\left(\frac{\pi c}{a+b}\right)$

- (1) $\frac{1+y^2}{1-y^2}$ (2) $\frac{2y}{1+y^2}$ (3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{y}{1+y^2}$

Ans. (3)

Sol. Let $\sin^{-1} x = a\lambda$, $\cos^{-1} x = b\lambda$, $\tan^{-1} y = c\lambda$

$$\Rightarrow (a+b)\lambda = \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{a+b} = 2\lambda$$

$$\text{Now } \cos\left(\frac{\pi}{a+b}\right) = \cos(2\lambda c) = \cos(2 \tan^{-1} y)$$

$$= \frac{1-y^2}{1+y^2}$$

12. If $\vec{a}$ & $\vec{b}$ are perpendicular vectors, then $\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b}))) =$

- (1) $|\vec{a}|^4 \vec{b}$ (2) $-|\vec{a}|^4 \vec{b}$ (3) $|\vec{a}|^2 \vec{b}$ (4) $-|\vec{a}|^2 \vec{b}$

Ans. (1)

Sol. $\vec{a} \times (\vec{a} \times ((\vec{a} \cdot \vec{b}) \vec{a} - |\vec{a}|^2 \vec{b}))$

$$= \vec{a} \times (-|\vec{a}|^2 (\vec{a} \times \vec{b})) = -|\vec{a}|^2 ((\vec{a} \cdot \vec{b}) \vec{a} - |\vec{a}|^2 \vec{b}) = -|\vec{a}|^4 \vec{b} - |\vec{a}|^2 (\vec{a} \cdot \vec{b}) \vec{a}$$

$$= |\vec{a}|^4 \vec{b} \quad (\because \vec{a} \cdot \vec{b} = 0)$$

13. If $|f(x) - f(y)| \leq |x - y|^2$; $x, y \in \mathbb{R}$ and $f(0) = 1$ then

- (1) $f(x) = 0$ for $x \in \mathbb{R}$ (2) $f(x) > 0$: $x \in \mathbb{R}$
(3) $f(x) < 0$; $x \in \mathbb{R}$ (4) $f(x)$ can take any value

Ans. (2)

Sol. $\left| \frac{f(x) - f(y)}{x - y} \right| \leq |x - y|$

$$\Rightarrow |f(x)| \leq 0$$

$$\Rightarrow f(x) = 0$$

$$\Rightarrow f(x) = \text{constant}$$

$$\Rightarrow f(x) = 1$$

14. $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \dots \infty$

(1) $\frac{13}{4}$

(2) $\frac{13}{2}$

(3) $\frac{11}{4}$

(4) $\frac{11}{2}$

Ans. (1)

Sol. $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \dots \infty$

(i)

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \dots$

(ii)

(i) - (ii)

$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \dots$

$\frac{2}{3}S = \frac{4}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \dots$

$\frac{2}{3}S = \frac{4}{3} + \frac{\frac{5}{3^2}}{1 - \frac{1}{3}} = \frac{4}{3} + \frac{5}{6} = \frac{13}{6}$

$S = \frac{13}{6} \times \frac{3}{2} = \frac{13}{4}$

15. Find maximum value of term independent of t in expansion of $\left(tx^{1/5} + \frac{(1-x)^{1/10}}{t} \right)^{10}$

(1) $56\sqrt{3}$

(2) $\frac{56}{\sqrt{3}}$

(3) 56

(4) $28\sqrt{3}$

Ans. (1)

Sol. $T_{r+1} = {}^{10}C_r (tx^{1/5})^{10-r} \left(\frac{(1-x)^{1/10}}{t} \right)^r$

$10 - r - r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5 x(1-x)^{1/2}$

$\frac{d(T_6)}{dx} = {}^{10}C_5 \left((1-x)^{1/2} + \frac{-x}{2\sqrt{1-x}} \right) = 0$

$2(1-x) - x = 0 \Rightarrow x = \frac{2}{3}$

Maximum $T_6 = {}^{10}C_5 \frac{2}{3} \left(\frac{1}{3} \right)^{1/2} = 56\sqrt{3}$

16. $\sum_{n=1}^{n=100} \int_{n-1}^n e^{x-[x]} dx$ is equal to :

- (1) $100(e-1)$ (2) $100e$ (3) 100 (4) $100(1-e)$

Ans. (1)

Sol. $\sum_{n=1}^{n=100} \int_{n-1}^n e^{\{x\}} dx$
 $= 100 \int_0^1 e^x = 100(e-1)$

17. If a fair coin is tossed n times, probability of getting 9 heads is equal to probability of getting 7 heads. Find the probability of given 2 heads.

- (1) ${}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$ (2) ${}^{16}C_2 \times \left(\frac{1}{2}\right)^{14}$ (3) ${}^{16}C_3 \times \left(\frac{1}{2}\right)^{16}$ (4) ${}^{16}C_3 \times \left(\frac{1}{2}\right)^{14}$

Ans. (1)

Sol. ${}^nC_9 \times \left(\frac{1}{2}\right)^9 \times \left(\frac{1}{2}\right)^{n-9} = {}^nC_7 \times \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^{n-7}$

${}^nC_9 = {}^nC_7 \Rightarrow n = 16$

$P(2\text{Heads}) = {}^{16}C_2 \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{14}$

$= {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$

18. Given three planes $P_1 : 3x - 15y + 21z = 9$

$P_2 : 4x - 20y + 21z = 10$

$P_3 : 2x - 10y + 14z = 10$

Then

- (1) P_1, P_2 are parallel (2) P_1, P_2, P_3 are parallel
 (3) P_1, P_3 are parallel (4) P_2, P_3 are parallel

Ans. (3)

Sol. $P_1 : x - 5y + 7z = 3$

$P_2 : 4x - 20y + 21z = 10$

$P_3 : x - 5y + 7z = 5$

P_1 and P_3 are parallel as dr's of normal are same

19. The summation of 2nd & 6th terms of an increasing GP is $\frac{25}{2}$ and product of 3rd & 5th term is 25, then summation of 4th, 6th & 8th term is
(1) 30 (2) 35 (3) 20 (4) 22

Ans. (2)

Sol. $ar + ar^5 = \frac{25}{2}$ and $ar^2 \cdot ar^4 = 25 \Rightarrow ar^3 = 5$

$$\therefore \frac{r+r^5}{r^3} = \frac{5}{2}$$

$$\Rightarrow 2 + 2r^4 = 5r^2$$

$$\Rightarrow 2r^4 - 5r^2 + 2 = 0$$

$$\Rightarrow r^2 = 2 \text{ or}$$

$$r^2 = \frac{1}{2} \text{ Reject}$$

$$\text{Now, } ar^3 + ar^5 + ar^7 = 5 + ar^5(1 + r^2) = 5 + 5 \cdot 2(1 + 2) = 35$$

20. If P(1,5,35) Q(7,5,2) R(1,λ,7) S(2λ,1,2) are coplanar then sum of value of λ is :

- (1) $\frac{39}{5}$ (2) $\frac{17}{2}$ (3) $\frac{-39}{5}$ (4) $\frac{-17}{2}$

Ans. (2)

Sol. for points to be coplanar
$$\begin{vmatrix} 6 & 0 & -33 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -38 \end{vmatrix} = 0$$

$$\Rightarrow 6(-33\lambda + 165 - 112) + 33(2\lambda^2 - 11\lambda + 5) = 0$$

$$\Rightarrow -198\lambda + 318 + 66\lambda^2 - 363\lambda + 165 = 0$$

$$\Rightarrow 66\lambda^2 - 561\lambda + 483 = 0$$

$$\text{Sum} = \frac{561}{66} = \frac{187}{22} = \frac{17}{2}$$

21. $\int_0^{\pi} |\sin 2x| dx$ is equal to

Ans. 2

Sol. $\int_0^{\pi} |\sin 2x| dx$

$$\text{Here } f(2a - x) = f(x)$$

$$= 2 \int_0^{\pi/2} (\sin 2x) dx$$

$$= 2 \left(-\frac{\cos 2x}{2} \right)_0^{\pi/2}$$

$$= 2$$

22. If $30 \cdot {}^{30}C_0 + 29 \cdot {}^{30}C_1 + 28 \cdot {}^{30}C_2 + \dots + {}^{30}C_{29} = n \cdot 2^m$ then find the value of $(m + n)$

Ans. 59

Sol. General term $= (30 - r) \cdot {}^{30}C_r$

$$\text{L.H.S} = \sum_{r=0}^{30} (30 - r) \cdot {}^{30}C_r$$

$$= 30 \sum_{r=0}^{30} {}^{30}C_r - \sum_{r=0}^{30} r \cdot {}^{30}C_r$$

$$= 30 \cdot 2^{30} - 30 \cdot 2^{29}$$

$$= 30 \cdot 2^{29}$$

$$\text{So } n = 30, m = 29$$

$$m + n = 59$$

23. If $x^3 - 2x^2 + 2x - 1 = 0$ has roots α, β, γ then find $(\alpha^{162} + \beta^{162} + \gamma^{162})$

Ans. 3

Sol. $x = 1, x = -\omega, x = -\omega^2$

$$\alpha = 1, \beta = -\omega, \gamma = -\omega^2$$

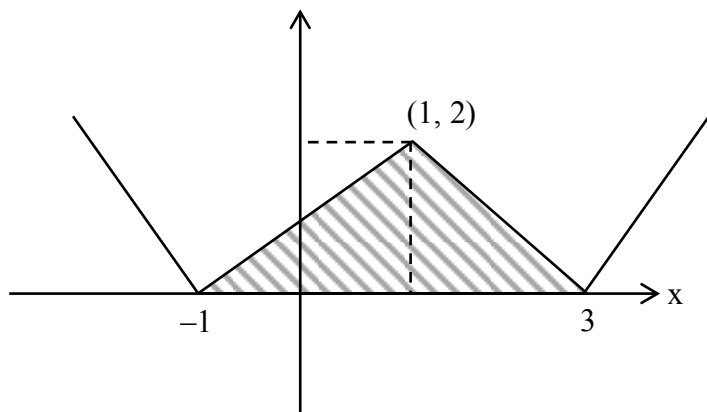
$$E = 1 + \omega^{162} + (\omega^2)^{162}$$

$$= 3$$

24. Find the area bounded by the curve $y = ||x - 1| - 2|$ with x-axis

Ans. 4

Sol.



$$\text{Area} = \frac{1}{2} \times 4 \times 2 = 4$$

25. Find number of solutions of $\sqrt{3} \cos^2 x = (\sqrt{3} - 1) \cos x + 1$ in $x \in \left[0, \frac{\pi}{2}\right]$

Ans. 1

Sol. $\sqrt{3} \cos^2 x - (\sqrt{3} - 1) \cos x - 1 = 0$

$$\cos x = \frac{(\sqrt{3}-1) \pm \sqrt{(\sqrt{3}-1)^2 + 4\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{(\sqrt{3}-1) \pm \sqrt{(4+2\sqrt{3})}}{2\sqrt{3}} = \frac{(\sqrt{3}-1) \pm (\sqrt{3}+1)}{2\sqrt{3}}$$

$$= 1, \frac{-1}{\sqrt{3}}$$

since $x \in \left[0, \frac{\pi}{2}\right]$

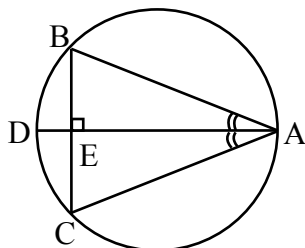
$$\Rightarrow \cos x = \frac{-1}{\sqrt{3}}, \text{ not possible}$$

$$\therefore \cos x = 1$$

$$\Rightarrow x = 0$$

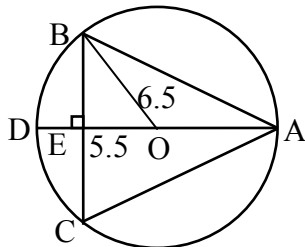
$\therefore$ number of solution 1

26. In the given figure $AD = 13$, $DE = 1$, AD bisects angle BAC and BC is perpendicular to AD , then, area of triangle ABC .



Ans. 41.568

Sol. Let O be mid-point of AD, now perpendicular from C to BC bisects chord BC, ($\triangle ACE$ and $\triangle ABE$ are congruent). Hence AD is diameter and O is centre of circle.



$$\text{So } BE = \sqrt{(6.5)^2 - (5.5)^2}$$

$$= \sqrt{12}$$

$$\text{Hence area} = \frac{1}{2} \cdot 12 \cdot 2\sqrt{12} = 24\sqrt{3}$$

27. Find the difference between the value of degree and order of differential equation corresponding to the family of curves $y^2 = a(x + \sqrt{a})$.

Ans. 2

Sol. order of differential equation is 1.

$$2yy' = a$$

$$\Rightarrow y^2 = 2yy' (x + \sqrt{2yy'})$$

$$\Rightarrow y - 2xy' = 2y' \cdot \sqrt{2yy'}$$

$$\Rightarrow (y - 2xy')^2 = 4(y')^2 \cdot 2yy'$$

$$\Rightarrow \left(y - 2x \cdot \frac{dy}{dx} \right)^2 = 8y \cdot \left(\frac{dy}{dx} \right)^3$$

Degree of Differential equation = 3

28. A plane is passing through $(\lambda, 2, 1)$ & $(4, -2, 2)$. It is perpendicular to line joining points

$$A(-2, 23, 18) \text{ and } B(-1, 29, 16). \text{ Find value of } \left(\frac{\lambda}{11} \right)^2 - \frac{4\lambda}{11} - 4$$

Ans. 8

Sol. $\vec{AB} = \hat{i} + 6\hat{j} - 2\hat{k}$

$$\vec{\alpha} = (\lambda - 4) \hat{i} + 4\hat{j} - \hat{k}$$

$$\vec{AB} \cdot \vec{\alpha} = 0$$

$$\lambda - 4 + 24 + 2 = 0 \Rightarrow \lambda = -22$$

$$E = 4 + 8 - 4 = 8$$

29. Number of bacteria are increasing at a rate proportional to its number at time 't' of at $t = 0$,

$N = 1000$ and after 2 hours, number of bacteria increased by 20%. If at $t = \frac{k}{\ln \frac{5}{6}}$, number of

bacteria are 2000, then find $\left(\frac{k}{\ln 2} \right)^2$?

Ans. 4

Sol. $\frac{dx}{dt} \propto x$

$$\Rightarrow \frac{dx}{dt} = \lambda x$$

$$\Rightarrow \int_{1000}^x \frac{dx}{x} = \lambda \int_0^t dt$$

$$\Rightarrow \ln \frac{x}{1000} = \lambda t$$

at $t = 2$, $x = 1200$

$$\therefore 2\lambda = \ln \frac{6}{5}$$

$$\therefore x = 1000 \cdot e^{\frac{1}{2} \ln \frac{6}{5} t}$$

Now $2000 = 1000 \cdot e^{\frac{1}{2} \ln \frac{6}{5} \cdot \frac{k}{\ln \frac{5}{6}}}$

$$\Rightarrow 2 = e^{\frac{k}{2}}$$

$$\Rightarrow \frac{k}{2} = -\ln 2$$

$$\Rightarrow \frac{k}{\ln 2} = -2$$